



The Scottish Mathematical Council

www.scot-maths.co.uk

MATHEMATICAL CHALLENGE 2013–2014

Entries must be the unaided efforts of individual pupils.

Solutions must include explanations and answers without explanation will be given no credit.

Do not feel that you must hand in answers to all the questions.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE

The Edinburgh Mathematical Society, The Maxwell Foundation, Professor L E Fraenkel,

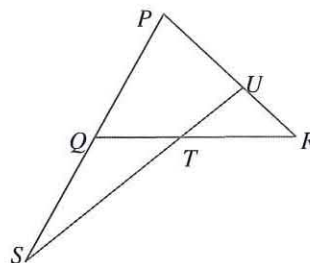
The London Mathematical Society and The Scottish International Education Trust.

The Scottish Mathematical Council is indebted to the above for their generous support and gratefully acknowledges financial and other assistance from schools, universities and education authorities.

Particular thanks are due to the Universities of Aberdeen, Edinburgh, Glasgow, Heriot Watt, Stirling, Strathclyde and to Preston Lodge High School, Bearsden Academy, Beaconsfield School and Northfield Academy.

Senior Division: Problems 2

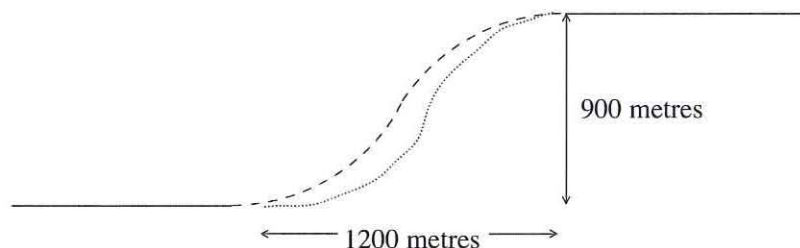
- S1. PQR is any triangle. The side PQ is extended to S where $PQ = QS$. The point U divides the side PR in the ratio $3 : 2$. The point T is where the lines QR and SU cross. Find the ratio $\frac{QT}{QR}$.



- S2. Dots are arranged in a rectangular grid with 4 rows and n columns. Consider different ways of colouring the dots, in which each dot either red or blue. A colouring is 'good' if no four dots of the same colour form a rectangle with horizontal and vertical sides. Find the maximum value of n for which there is a good colouring.

- S3. A triangle has sides of length 4, 7 and 9 units. Find the length of the longest median. **Show your reasoning.**

S4.



Two straight sections of a road, each running from east to west, and located as shown, are to be joined smoothly by a new roadway consisting of arcs of two circles of equal radius. The existing roads are to be tangents at the joins and the arcs themselves are to have a common tangent where they meet. Find the length of the radius of these arcs.

SEE OVER FOR QUESTION S5.



Mathematical Challenge Problems 2

SENIOR DIVISION 2013–2014

PLEASE USE CAPITALS TO COMPLETE

SURNAME

OTHER NAME(S)
(underline the one
you prefer)

SCHOOL

AGE

YEAR OF STUDY

S

FOR OFFICIAL USE

Marker

Marks

1	2	3	4	5

Total

— — — — - CUT ALONG HERE — — — —

Please write your solutions on A4 paper and staple the above form to them.

PLEASE WRITE YOUR NAME ON EVERY PAGE.

Send your entry directly or through your school to :

Ruth Forrester,

University of Edinburgh, Moray House Institute of Education,
Holyrood Rd., Edinburgh, EH8 8AQ

For further information on the competition, please see the Information Circular, which has been distributed to all secondary schools. Please contact the local organiser, whose name and address are given above, if you require a further copy.

S5. Let

$$f(x) = x^n + a_1x^{n-1} + \dots + a_n,$$

where $a_1, a_2, \dots, a_n$ are given numbers. It is given that $f(x)$ can be written in the form

$$f(x) = (x + k_1)(x + k_2) \dots (x + k_n).$$

By considering $f(0)$, or otherwise, show that $k_1 k_2 \dots k_n = a_n$.

Show also that

$$(k_1 + 1)(k_2 + 1) \dots (k_n + 1) = 1 + a_1 + a_2 + \dots + a_n$$

and give the corresponding result for $(k_1 - 1)(k_2 - 1) \dots (k_n - 1)$.

Hence, find the roots of the equation

$$x^4 + 22x^3 + 172x^2 + 552x + 576 = 0,$$

given that they are all integers.

END OF PROBLEM SET 2

CLOSING DATE FOR RECEIPT OF SOLUTIONS :

14th February 2014

Look on the SMC web site: www.scot-maths.co.uk for information about Mathematical Challenge